Teaching Workshop:
The language we use to teach mathematics

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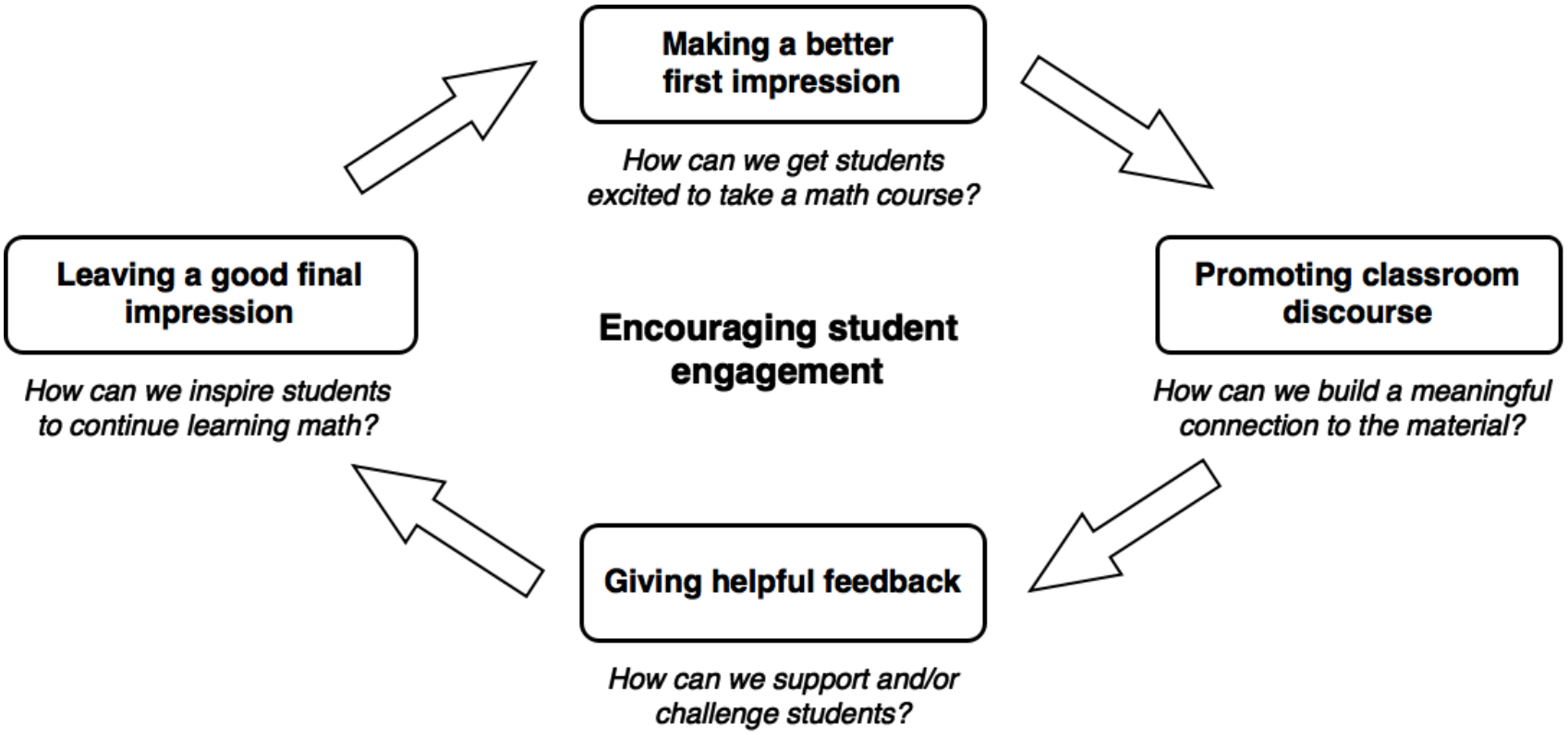
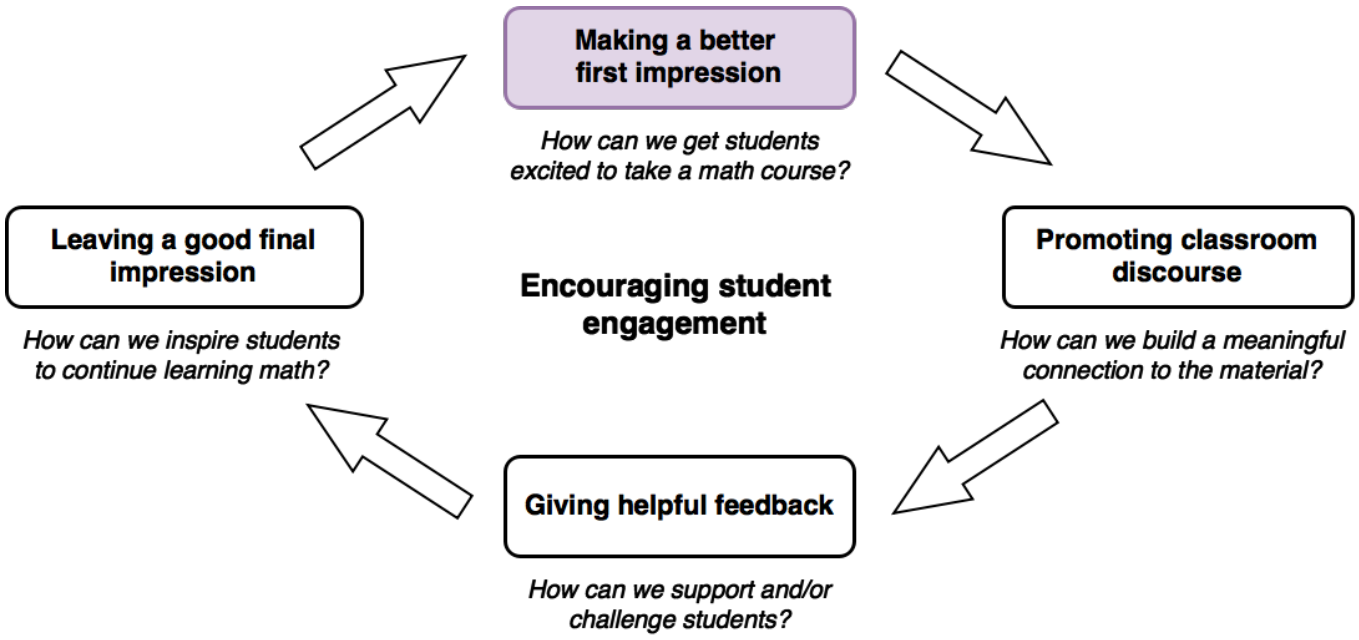
How can we help our students succeed?



Part I:
Using accessible language for new students



Starting off on the wrong foot

- Many students enrol in a first-year math course simply because it is required for their program.
- These students may view mathematics as a collection of arbitrary laws and formulae with no purpose and limited applications to real situations.
- Bombarding students with new terms, definitions, equations, and theorems only makes this problem worse.
- We need to step back and think about the language we use and the impact it has on student motivation.

How does math sound to a new student?

Mathematics 1229: Methods of Matrix Algebra

Description: A course in matrix algebra intended primarily for students in the Social Sciences.

Course Outline: Vectors in $\mathbb{R}^m$; Equations of lines and planes; Linear Equations; Solution of Linear Systems; Matrix Algebra; Matrix Multiplication and Inverses; Determinants.

What information does this provide to an incoming student?

What might a student assume they will be asked to do?

How much of this will only make sense *after* taking the course?

What students need to know

What will I SEE in this course?

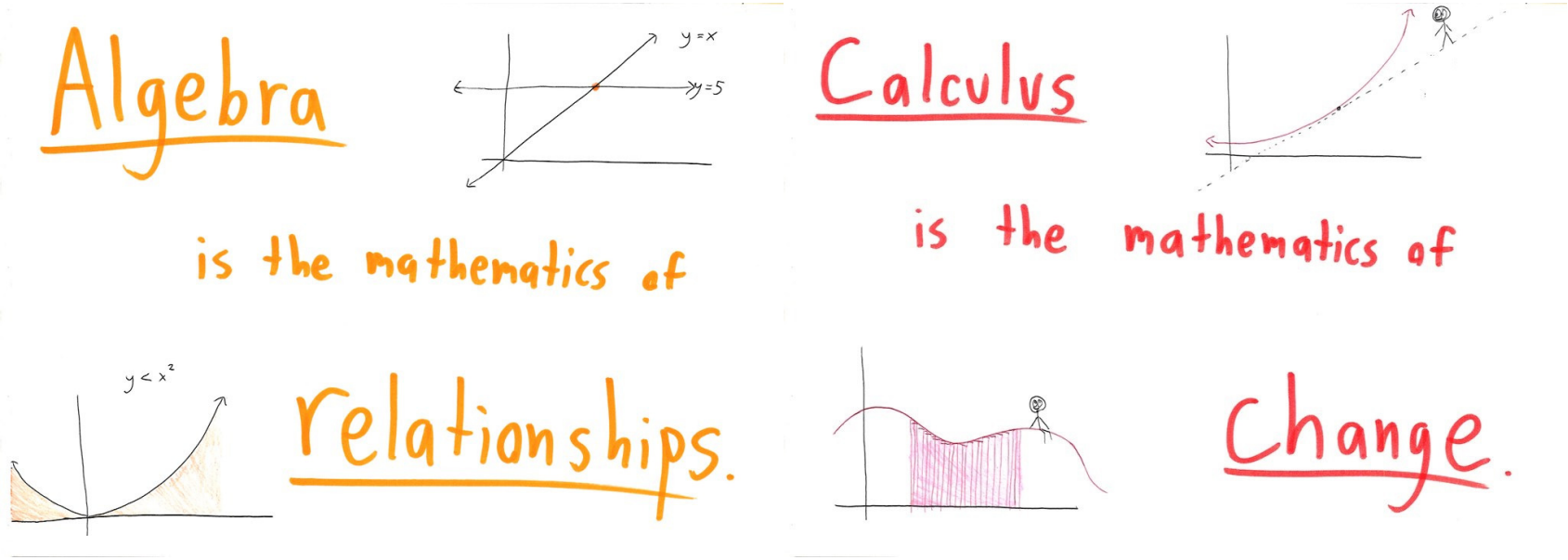
- vectors in $\mathbb{R}^m$: *a language to describe position and orientation in multi-dimensional space*
- solutions of linear systems: *a formal way to define the relationships among a set of lines and planes*
- matrix algebra: *a type of arithmetic used for transforming lines and planes in multi-dimensional space*

What will I DO in this course?

- Develop a new language for describing and expanding on familiar geometric ideas.
- Find patterns and make conjectures.
- Provide logical justifications which support true statements.
- Construct counter examples to disprove false statements.

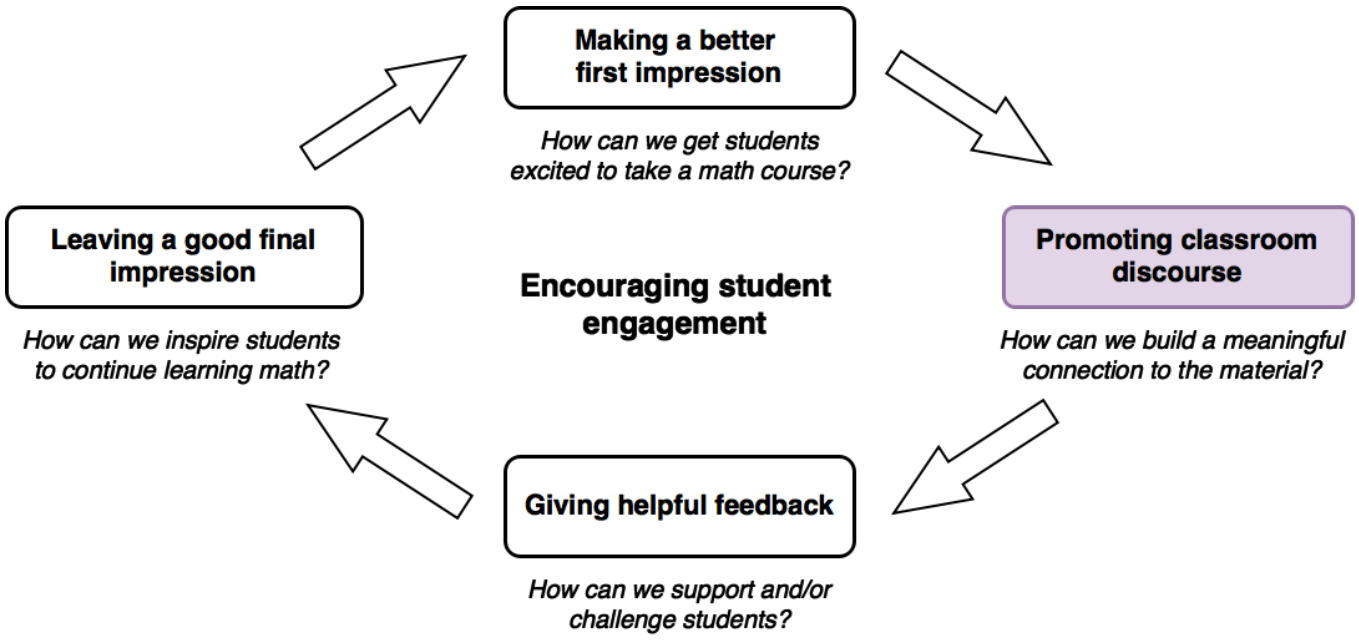
Small group activity

1. Read your group's student profile and course description.
2. Identify any language which may be inaccessible or unhelpful to the student.
3. Propose translations and/or additions which create useful information for the student.



Illustrations by Ben Orlin: https://mathwithbaddrawings.com/2015/05/27/one-word-math-classes/

Part II:
Engaging students with mathematical ideas



Two Discursive voices

- **Monogloss voice:**
  - provides facts or “bare assertions”.
  - sounds impersonal, descriptive, or report-like.
  - does not seek to engage the listener/reader.
  - *is used to present the standardized forms of mathematical constructs.*
- **Heterogloss voice:**
  - grounds a proposition in the personal subjectivity of the speaker/writer.
  - leaves room for negotiation or re-examination.
  - engages the listener/reader at different levels.
  - *is used to convey the diverse expressions of mathematical meaning.*

Monogloss vs. heterogloss

Consider the following monoglossic proposition:

> There exists no elementary antiderivative for the Gaussian function, which is defined by $f(x) = e^{-x^2}$.

- provides no opportunity for consideration or conjecture
- may be viewed as an arbitrary fact to memorize

Consider the following alternative:

> Computing an antiderivative for e^{-x^2} looks like it should be straightforward. It turns out, however, that this function has no elementary antiderivative at all. We call the function $f(x) = e^{-x^2}$ the *Gaussian function*.

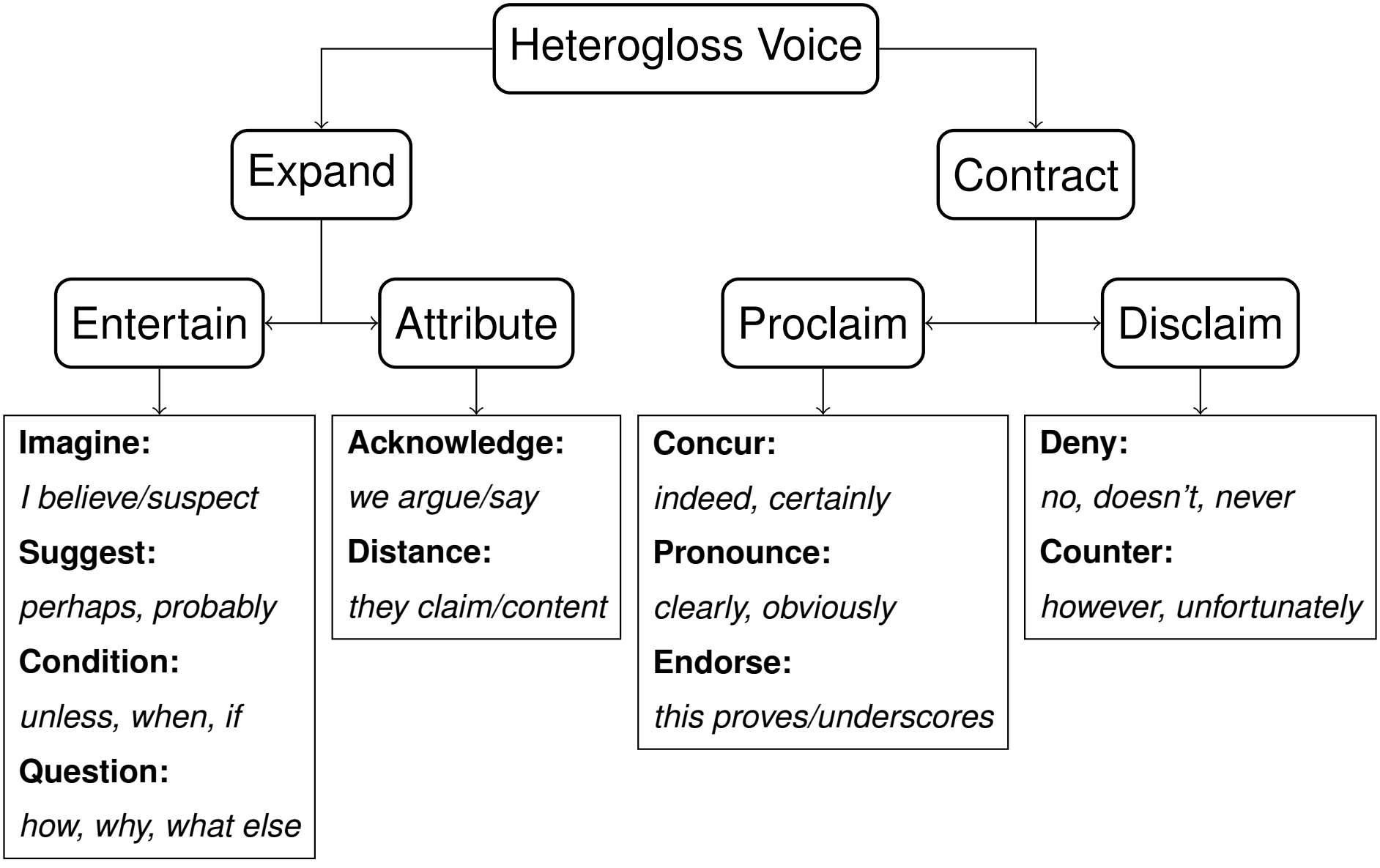
Why include heterogloss voice?

1. Invite students to consider possible strategies for solving the problem.

Computing an antiderivative for e^{-x^2} looks like it should be straightforward. It turns out, however, that this function has no elementary antiderivative at all. We call the function $f(x) = e^{-x^2}$ the Gaussian function.

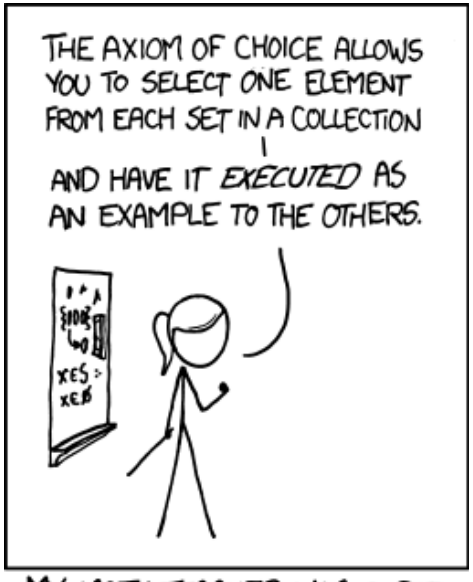
2. Counter any such strategies by denying their chance of success.

3. Acknowledge the importance of this function.



Adapted from Mesa, V., & Chang, P. (2010). The language of engagement in two highly interactive undergraduate mathematics classrooms, *Linguistics and Education, 21* (2), 83-100.

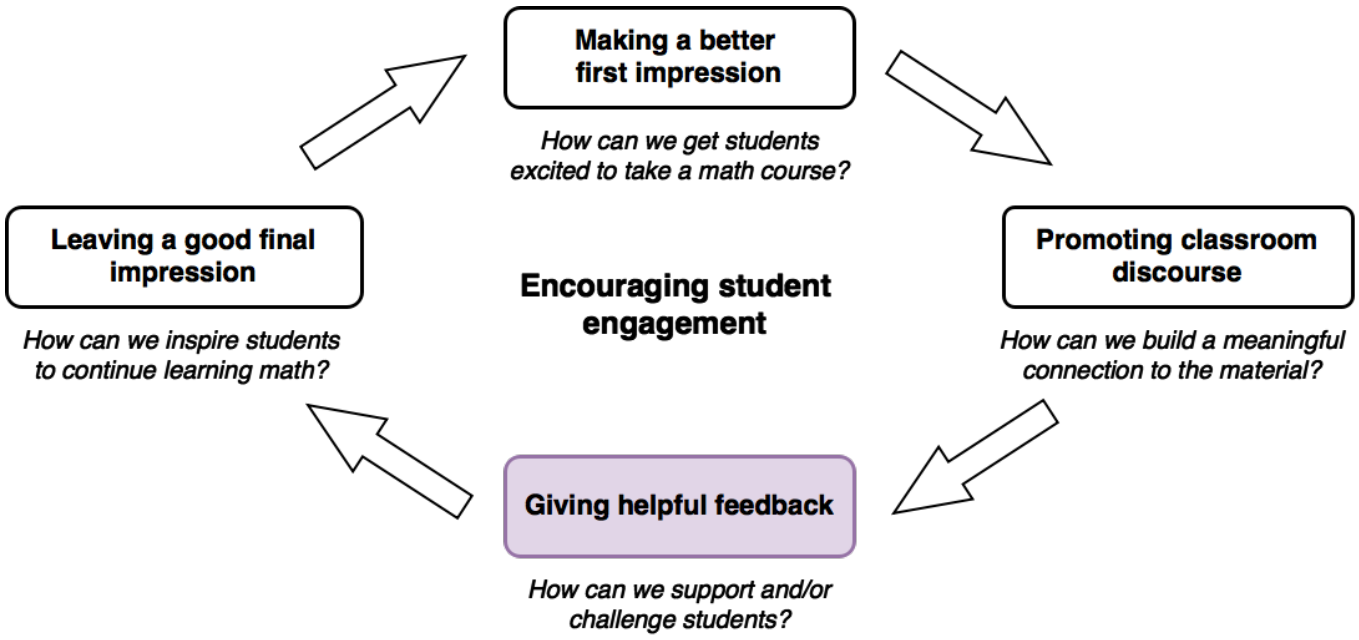
How can we use the heterogloss voice?



1. How can we use heterogloss statements or questions to encourage deeper thinking?
2. Can we use heterogloss statements or questions to assess students' understanding?
3. Do we need to include monogloss statements and/or questions when teaching?
4. Can we include heterogloss statements and questions and still cover all the course material in time?

Illustration by Randall Munroe: https://xkcd.com/982/

Part III:
Providing feedback and support to students



Individual activity

1. Browse the examples of feedback posted around the room.
2. Place one or more sticky dots on each example that you would consider **helpful** to students.
3. More dots = more helpful; use as many as you'd like.

Illustration by Jorge Cham: http://www.phdcomics.com/comics.php?f=1510

Good intentions

“It’s ok - not everyone can be good at math.”

- One view of mathematical intelligence is that it is a fixed attribute, dictated by genetics and unchangeable
- A student who believes him or herself to be “just bad at math” will have decreased motivation, learning outcomes, and achievement.
- When an instructor views a student as low-ability, they are more likely to offer “comforting” feedback.

“This stuff doesn’t come naturally to everybody.”

“Your strengths just lie in other subjects.”

“Maybe an easier question would be more on your level.”

Feedback that backfires

- When students receive comfort-oriented feedback, they perceive their instructor/TA as having lower expectations and less investment in their success [1].
- They report lowered motivation themselves and expect to receive a lower final grade in the course [1].

Warning:

- Even if our feedback is given with the best intentions, the way we express support can ultimately backfire.
- By *avoiding* comforting statements, we can actually help a struggling student succeed.

[1] Rattan, A., Good, C., & Dweck, C.S. (2012). “It’s ok - Not everyone can be good at math”: Instructors with an entity theory comfort (and demotivate) students, emphJournal of Experimental Social Psychology, 48 (3), 731-737.

A better alternative

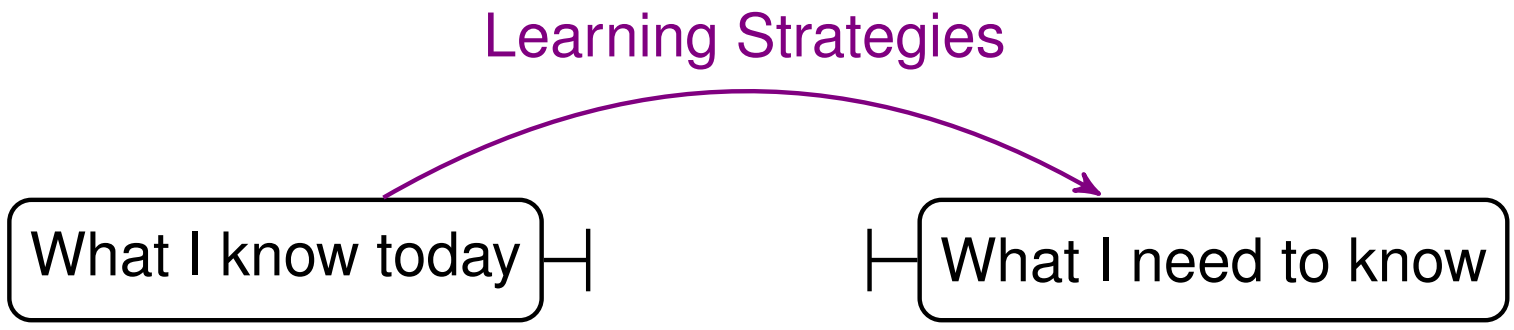
“I can see that you are working hard to learn these concepts.”

“I understand that you probably have a lot going on in your other courses right now.”

- This feedback is still “kind”, but does not make any assessment of the student’s mathematical ability
- These statements are called *statements of caring*.
- Statements of caring can be seen as natural component of positive feedback.
- They help to build a relationship with the student and can increase their motivation, but do not directly affect the student’s learning outcomes.

The goal of feedback

The primary goal of feedback is to bridge the gap between current understanding and expected learning outcomes.



By giving strategy-oriented feedback, we provide students with concrete tools for building this bridge.

Students who receive strategy-oriented feedback have more positive perceptions of their instructor's expectations and investment [1].

[1] Rattan, A., Good, C., & Dweck, C.S. (2012). "It's ok - Not everyone can be good at math": Instructors with an entity theory comfort (and demotivate) students, emphJournal of Experimental Social Psychology, 48 (3), 731-737.

Strategy-oriented feedback

Useful learning strategies include:

1. breaking down problems, relating new material to old (cognitive - how to think)
2. goal-setting, self-assessing, test debriefing (metacognitive - how to evaluate your thinking)
3. working with others, developing a study plan (effort management - how to work)

The use of effort management strategies is correlated to motivation and is a significant predictor of achievement [2].

[2] Pokay, P., & Blumenfeld, P.C. (1990). Predicting achievement early and late in the semester: The role of motivation and use of learning strategies, *Journal of Educational Psychology, 82* (1), 45-50.

“Can you just show me how to do this problem?”

Show me what you’ve tried so far, and why it didn’t work. ✓✓

“I don’t even know how to start these questions.”

Assemble a “cheat sheet” for the course. ✓✓
(list of derivative rules, named theorems, definitions, examples, etc.)

“How did they get from this line to the next one?”

Identify and fill any gaps in prior knowledge or core skills. ✓✓
(e.g. trig identities, exponent laws, fractions)

“How do I know if I’m ready for the exam?”

Complete practice tests in a closed-book, timed environment. ✓✓

“But in the solutions manual...”

Use the solutions manual ONLY for checking completed solutions.
It is not the place to learn concepts or procedures.✓✓✓

Thank You!

All information and resources will be posted at carolinejunkins.com/teaching-workshop.html

*This workshop is eligible for Future Prof credit with the TSC. Please see me for details.