Transformations of the Logarithmic Function

Again, we will summarize the transformations that apply to the graph of $y = \log_b x$, which you could be asked to sketch.

Given the graph of a logarithmic function $y = \log_b x$, a **transformation** of this graph is given by a function of the form

$$f(x) = M \log_b(Dx + h) + k$$

where, M, D, h, and k are real numbers.

M determines the **vertical reflection** and **vertical stretch** of the graph.
If $M < 0$, the graph is reflected about the x-axis (y-values become negative).
If $0 < |M| < 1$, the graph is compressed vertically by a factor of M.
If $|M| > 1$, the graph is expanded vertically by a factor of M.
(In any of these cases, each y-coordinate is multiplied by M)

D determines the **horizontal reflection** and **horizontal stretch** of the graph.
If $D < 0$, the graph is reflected about the y-axis (x-values become negative).
If $0 < |D| < 1$, the graph is expanded horizontally by a factor of D.
If $|D| > 1$, the graph is compressed horizontally by a factor of D.
(In any of these cases, each x-coordinate is divided by D)

k determines the **vertical translation** of the graph.
If $k > 0$, the graph is shifted up by k units.
If $k < 0$, the graph is shifted down by k units.
(In either case, k is added to each y-coordinate)

h determines the **horizontal translation** of the graph.
If $h > 0$, the graph is shifted left by h units.
If $h < 0$, the graph is shifted right by h units.
(In either case, h is subtracted from each x-coordinate)

To graph a logarithmic function of this form, we proceed in the following way:

1. List the reference function $y = \log_b x$ and all applicable transformations (to determine how the graph should look).
2. Find the new location of the "special point" (originally at $(1, 0)$).
   For the x-coordinate: Solve the equation $Dx + h = 1$ for x.
   For the y-coordinate: $y = k$.
3. Find the new location of the vertical asymptote: $x = \dfrac{-h}{D}$.
4. Plot the special point and asymptote on your xy-plane. Using the transformations and limits as a guide, draw the curve $y = f(x)$.