Transformations of the Exponential Function

Given the graph of an exponential function $y = b^x$, a **transformation** of this graph is given by a function of the form

$$f(x) = Mb^{Dx+h} + k$$

where, M, D, h, and k are real numbers.

M determines the **vertical reflection** and **vertical stretch** of the graph.
If $M < 0$, the graph is reflected about the x-axis (y-values become negative).
If $0 < |M| < 1$, the graph is compressed vertically by a factor of M.
If $|M| > 1$, the graph is expanded vertically by a factor of M.
(In any of these cases, each y-coordinate is multiplied by M)

D determines the **horizontal reflection** and **horizontal stretch** of the graph.
If $D < 0$, the graph is reflected about the y-axis (x-values become negative).
If $0 < |D| < 1$, the graph is expanded horizontally by a factor of D.
If $|D| > 1$, the graph is compressed horizontally by a factor of D.
(In any of these cases, each x-coordinate is divided by D)

k determines the **vertical translation** of the graph.
If $k > 0$, the graph is shifted up by k units.
If $k < 0$, the graph is shifted down by k units.
(In either case, k is added to each y-coordinate)

h determines the **horizontal translation** of the graph.
If $h > 0$, the graph is shifted left by h units.
If $h < 0$, the graph is shifted right by h units.
(In either case, h is subtracted from each x-coordinate)

To graph an exponential function of this form, we proceed in the following way:

1. List the reference function $y = b^x$ and all applicable transformations (to determine how the graph should look).

2. Find the new location of the "special point" (originally at $(0, 1)$).
   For the x-coordinate: Solve the equation $Dx + h = 0$ for x.
   For the y-coordinate: Solve the equation $y = M + k$ for y.

3. Find the new location of the horizontal asymptote (originally at $y = 0$).
   Compute the limits
   $$\lim_{x\to\infty} f(x) \text{ and } \lim_{x\to-\infty} f(x)$$
   Precisely one of these limits will give you a real number a. The horizontal asymptote is located at $y = a$. The other limit will tell you if the function is increasing or decreasing.

4. Plot the special point and asymptote on your xy-plane. Using the transformations and limits as a guide, draw the curve $y = f(x)$.