Teaching Portfolio

Caroline Junkins

Contents

Statement of Teaching Philosophy **2**

Teaching Responsibilities **4**
- Instructor, Western University . . . 4
- Teaching Assistant, University of Ottawa . . . 5
- Teaching Assistant & Help Centre Supervisor, McMaster University . . . 5

Evidence of Teaching Effectiveness **6**
- Teaching Evaluations: Mean Scores . . . 6
- Teaching Evaluations: Selected Comments . . . 6
- Student Letters of Support . . . 7

Teaching Innovations **8**
- Collaborative Learning Activities . . . 8
- Flowcharts . . . 8

Educational Leadership **9**
- CMS Special Session on Math Education . . . 9
- Teaching Workshop . . . 9
- CMS Notes Article . . . 9

Professional Development **10**
- Connecting Women in Mathematics Across Canada . . . 10
- Advanced Teaching Program . . . 10
- Fall Perspectives on Teaching Conference . . . 10

Appendix A: Collaborative Learning Activities **11**

Appendix B: Teaching Workshop **15**

Appendix C: Limit Flowcharts **18**

Statement of Teaching Philosophy

> *"It is good to express a thing twice right at the outset and so to give it a right foot and also a left one. Truth can surely stand on one leg, but with two it will be able to walk and get around."* (Friedrich Nietzsche)

To me, mathematics is surely a two-legged beast. The left leg is built from definitions, axioms, theorems, and rules, and provides the language required to discuss, prove, or disprove ideas. The right leg is formed by motivation, meaning, and intuition. It is the right leg that provokes questions, seeks to make connections, and provides context for the results described by the left. A student with strong mathematical legs can "get around" the world of mathematics, travelling and exploring independently.

My role as an instructor is to help my students strengthen both of their mathematical legs, and to model how the two can work most effectively together. In addition to ensuring that students have a solid understanding of the course material, my goal is to encourage students to communicate ideas effectively, relate new material to prior knowledge, apply mathematical techniques to various situations, and test the limitations of such techniques.

Teaching Strategies

By augmenting traditional lectures with well-chosen activities and assessments, we can achieve a balanced approach to learning. Recent research in cognitive science has indicated that students benefit most from explicit instruction which is enhanced by guided discovery (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). In my linear algebra course, lectures are accompanied by short quizzes and collaborative learning activities (Appendix A). The quizzes provide students with feedback on their mastery of the core skills required for the subject, with an emphasis on procedures and algorithms. The collaborative learning activities are designed to highlight concepts and connections, as well as to expand on ideas from class.

Research on large introductory microeconomics courses at Indiana University has shown that this type of collaborative learning activity is beneficial to students at both ends of the grade spectrum. Higher-achieving students show an improvement in a broad range of learning objectives, while students in the bottom 40 percentile show the most improvement in terms of exam performance (Huynh, Jacho-Chavez, & Self, 2015).

Personal Strengths

The qualities which help me to be an effective teacher are my enthusiasm, patience, and approachability. It is clear to my students that I love math and that I enjoy discussing mathematical ideas. Not all of my students share this sentiment, but my enthusiasm for the subject helps to promote an upbeat learning environment and can sometimes be contagious. I strongly believe that every student has the potential to succeed in their math courses, and that math should not be associated with helplessness or fear for anyone.

Many students lack the foundational skills required for university math courses and struggle to understand the material being taught in class. It is important to carefully work through these problems step-by-step, to find where and why a student is getting lost. For

example, I have found that some students struggle with applying rules of differentiation simply because they do not know how to work with exponents. Patience is essential when working with a student who considers themselves to be "bad at math", as they are more likely to feel overwhelmed and hopeless when encountering difficulty (Dweck, 1999).

By being approachable and available, am I able to reach these students and give them individualized help. Interacting with students outside the classroom also allows me to gauge how the class is learning and to understand the backgrounds and motivations of my students. Incorporating this information into my teaching helps me to develop a stronger relationship with my students and to present material in a way which is more meaningful to them.

Professional Development

My approach to teaching is evolving as I gain experience in the classroom and learn more about current research on teaching and learning. During my postdoctoral fellowship at Western University, I have participated in programs offered by the Teaching Support Centre. I have also designed and offered a teaching workshop, completed by 21 graduate students from the departments of Mathematics, Applied Mathematics, and Statistics (Appendix B).

Together with Prof. John McLoughlin from the University of New Brunswick, I am co-organizing a session on Math Education for the 2016 CMS Winter meeting. Our session *Teaching for Deeper Mathematical Engagement* will bring together teachers from various institutions to share their own methods and initiatives for engaging students with mathematical ideas. The goal of the session is to enhance awareness of pedagogical methods while acknowledging the challenges in place with teaching mathematics at the post-secondary level.

Career Goals

I seek a position which would allow me to conduct research on teaching and learning at the post-secondary level. I plan to develop, or contribute to, initiatives which encourage participation in mathematics by underrepresented groups such as women, minorities, and First Nations students. I am interested in creating resources which help first-generation students satisfy their mathematics requirement and foster a healthy attitude towards mathematics.

I strongly believe in evidence-based teaching practices, and that it is important to accurately assess how our teaching methods affect each member of our diverse student body. Students come to university with different backgrounds, different learning styles, and different motivations. It is my goal as an instructor to ensure that every student has the guidance and support to strengthen their mathematical legs before they walk out into the world.

References

Alfieri, L., Brooks, P. J., Aldrich, N. J., & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology, 103*(1), 1-18.

Dweck, C. S. (1999). *Self-theories: Their role in motivation, personality, and development.* Philadelphia, PA: Psychology Press.

Huynh, K. P., Jaco-Chavez, D. T., & Self, J. K. (2015). The distributional efficacy of collaborative learning on student outcomes. *American Economist, 60*(2), 98-119.

Teaching Responsibilities

2014-present: Instructor, Western University

Math 1228B: Methods of Finite Mathematics
Teaching: Winter 2017 (coordinated by V. Olds)
Number of students: 243

This course is a basic introduction to finite mathematics intended for students in the social sciences. Topics include techniques of counting, probability, discrete and continuous random variables. Students are expected to demonstrate an understanding of these concepts and an ability to apply them in solving a variety of problems. Student evaluation consists of written exams and online quizzes.

Math 1600A: Linear Algebra I
Teaching: Fall 2016 (coordinated by M. Pinsonnault)
Number of students: 170

Math 1600A is an introductory linear algebra course for students in mathematics, physical sciences, and economics. Course material includes: properties and applications of vectors, matrix algebra, solving systems of linear equations, determinants, vector spaces, orthogonality, eigenvalues and eigenvectors. Student evaluation consists of written exams, weekly quizzes, and collaborative learning activities.

Math 9403L: Topics in Algebraic Groups: Galois Cohomology
Facilitated: Summer 2016
Number of students: 1 graduate

In this graduate reading course we worked through sections of *The Book of Involutions* relating to Galois cohomology, torsors, and cohomological invariants. We then considered applications of these topics to projective homogeneous varieties such as generalized Severi-Brauer varieties. Student evaluation consisted of oral presentations and written exposition.

Math 3121A/9050A: Advanced Linear Algebra
Taught/Coordinated: Fall 2015
Number of students: 3 graduate, 4 undergraduate

This course is a continuation of the undergraduate linear algebra stream, including topics such as invariant subspaces, the Cayley-Hamilton theorem, primary decomposition theorem, Jordan canonical form, rational canonical form, inner product spaces, Gram-Schmidt orthogonalization, orthogonal projections and the spectral theorem. The course is cross-listed as a graduate course, satisfying requirements for both Masters and Doctoral students. Student evaluation consisted of written exams and assignments. Each assignment/exam included more challenging questions which were required for graduate students only. Undergraduate students were encouraged to solve these questions for bonus marks.

Calculus 1501B: Calculus II for Mathematical and Physical Sciences
Taught Winter 2015 (coordinated by J. Adamus)
Number of students: 101

Calculus 1501B is for students who intend to pursue a degree in Actuarial Science, Applied Mathematics, Astronomy, Mathematics, Physics, or Statistics. Course material includes techniques of integration, the Mean Value Theorem and its consequences, series, Taylor series with applications, parametric and polar curves with applications, first order linear and separable differential equations with applications. Student evaluation consisted of weekly online assignments (through WeBWorK), written assignments, and written exams.

Math 0110A: Introductory Calculus
Taught Fall 2015 & Fall 2014 (coordinated by V. Olds)
Number of students: 93 (in 2015), 87 (in 2014)

Incoming students who require a university level course in calculus in order to enter a module, but who did not complete introductory calculus in secondary school are required to first complete Mathematics 0110A. Course material includes limits, continuity, definition of derivative, rules for differentiation, implicit differentiation, velocity, acceleration, related rates, maxima and minima, exponential functions, logarithmic functions, differentiation of exponential and logarithmic functions, curve sketching. Student evaluation consisted of written assignments and exams.

2010-2012: Teaching Assistant, University of Ottawa

Responsibilities included facilitating tutorials, grading tests and assignments, substituting for lectures, proctoring exams, and providing one-on-one help to students during office hours.

MAT3341: Applied linear algebra I (Winter 2012)
MAT2141: Linear Algebra I (Fall 2011)
MAT1339: Introduction to Calculus and Vectors (Fall 2011)
MAT2143: Algebraic Structures (Winter 2011)
MAT1339: Introduction to Calculus and Vectors (Fall 2010)

2008-2010: Math Help Centre Supervisor, McMaster University

Responsibilities included supervising teaching assistants, organizing groups of students with similar problems and connecting them with appropriate teaching assistants, helping individual students, and running course-wide review sessions.

2008-2010: Teaching Assistant, McMaster University

Responsibilities included facilitating tutorials, grading tests and assignments, substituting for lectures, proctoring exams, and providing one-on-one help to students during office hours.

Math 3H03: Number Theory (Winter 2010)
Math 1F03: Introduction to Calculus and Analytic Geometry (Fall 2009)
Math 1X03: Calculus for Students in Mathematics and Statistics (Fall 2008)

Evidence of Teaching Effectiveness

Teaching Evaluations: Mean Scores

Category 7=outstanding 1=poor	**Math 0110** Fall 2015 49 responses	**Math 3121** Fall 2015 6 responses	**Calc 1501** Winter 2015 50 responses	**Math 0110** Fall 2014 42 responses
Displays Enthusiasm	6.6	6.7	6.5	6.3
Well Organized	6.8	6.8	6.4	6.5
Explains Concepts Clearly	6.4	6.3	6.1	6.1
Encourages Participation	6.0	6.0	5.5	5.7
Responses Clear and Thorough	6.7	6.3	6.3	6.3
Interesting Presentations	6.2	6.0	6.0	6.0
Shows Concern	6.4	6.0	6.2	6.4
Available for Consultation	6.6	6.3	6.3	6.6
Communicates Well	6.6	6.3	6.1	6.4
Adheres to Course Objectives	6.7	6.5	6.3	6.6
Relates Topics Well	6.6	6.7	6.1	6.3
Provides Fair Evaluations	6.6	6.3	5.5	6.4
Grades Work Promptly	6.4	6.8	5.9	6.4
Good Motivator	6.3	5.7	5.7	5.8
Average of 1-14	6.5	6.3	6.1	6.3
Overall Effectiveness	6.7	6.5	6.5	6.6
Course as a learning experience	6.4	5.5	5.9	5.8

Teaching Evaluations: Selected Comments

Math 0110A - Fall 2015:

Good enthusiasm about material. Very open to questions. Very good knowledge of material.

Professor Junkins is an excellent teacher. She's very patient when answering questions. She's also very good at explaining concepts clearly.

Very nice. Explains concepts clearly, organized, and patient.

Dr. Junkins, you bring an awe inspiring enthusiasm to calculus. Your approachable manner, easy going demeanour and ability to explain concepts go far to encourage students. Thank you for being a most wonderful teacher and good luck to you in future.

Math 3121/9050A - Fall 2015:

From the way she presented the material it was easy to see how everything fit together in the course. Her assistance in office hours was always very beneficial to me.

Carolyn is very organized. She grades assignments quickly and she responds to emails about the assignments quickly. Sometimes the lecture goes longer then it should.

Calc 1501B - Winter 2015:

Very clear about subject material and makes a dedicated effort to offer individual consultation regarding questions and material. Her class has been by far my best experience with mathematics and the applied math department.

Very fair, excellent at explaining concepts and providing help when necessary.

Professor Junkins is outstanding. The course material is extraordinarily difficult and she makes it simple. In class she never falters which makes it seem easy. She cares about her students and is there for you if you need help. She makes dry subjects interesting by talking about the history and other crucial aspects of math. Overall I am glad to have her as my Professor, and wish her all the best in the future.

Math 0110A - Fall 2014:

Dr. Junkins is a wonderful Professor who cares about student success. I've never understood Math so well.

Very clear and easy to understand explanation, and very good lecture notes. Also very enthusiastic and willing to help students. Great Prof. Thank you.

Prof. Junkins is always available to assist in her office hours and outside of that should you need it. I felt like she was always attempting to explain new topics in as many ways as possible until we understood.

C. Junkins seems to be very enthusiastic in teaching material. Particularly like terms used like "ugly problems" and "hideous problems" adds truth and humour to a very "hideous" class in the eyes of a non-mathematical thinker!

You're a super amazing Prof you explain things clearly, are incredible nice (with an occasional sense of humour thrown in!), and make the classroom genuinely feel like a fun place to learn. I don't want to do math for a profession, and I didn't want to take this course in the first place, but if it was any other Prof I'd definitely be failing by now. Thanks for a fun (if not incredibly hard) semester!

Student Letters of Support

Three former students have provided letters of recommendation addressing my effectiveness as an instructor. Each student was asked to provide an honest account of their experience and was assured that providing a letter would in no way affect our relationship or their future grades, should they ever enrol in one of my classes in the future.

Copies of all letters are available upon request.

Letter #1: An undergraduate student in Calculus 1501B, Winter 2015

Letter #2: A graduate student in Math 9403L, Summer 2016

Letter #3: An undergraduate student in Math 0110A, Fall 2015

Teaching Innovations

Collaborative Learning Activities

As part of my current Linear Algebra class, I have introduced collaborative learning activities which take place throughout the semester. Students work in groups of 4-5 to answer a series of questions designed to expand on concepts taught in lecture and highlight connections between different modules of course material. A teaching assistant facilitates the activity, ensuring that all group members are contributing to the discussion, and encouraging students to consider different approaches to each question.

The activities themselves are adapted from various sources, including the NSF-funded IOLA Study Team (Developing Inquiry-Oriented Instructional Materials for Linear Algebra Instruction: http://iola.math.vt.edu/index.php) and the MAA Innovative Teaching Exchange (http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/innovative-teaching-exchange).

In addition to helping students achieve the specific learning objectives of the course, these activities can have a positive effect on developing broader skills such as communicating mathematical ideas, forming conjectures, and writing proofs. Many students in this course intend to pursue majors in mathematics, applied mathematics, and actuarial science, where these skills will be invaluable for their future studies.

Flowcharts

In my "catch-up" calculus class Math 0110A, most students are encountering limits for the first time, and many struggle with this new concept, as it is more abstract than topics they've covered in previous math classes. I found that many students freeze when faced with a limit problem, unsure where to begin, or what it is they are trying to compute.

After seeing how much difficulty students were having with their practice problems, I developed 3 flowcharts which they could consult while working on limit problems, intended to guide them through the standard computation of the limit at a point or at infinity (see Appendix C). With flowcharts in hand, students had the confidence to successfully complete their practice problems. Once their anxiety surrounding limits had waned, I was able to return to the abstract concept of a limit and encourage students to consider different approaches and more challenging questions.

When teaching the course for the second time, I briefly introduced the concept of a limit, and then immediately provided students with the flowcharts and a set of practice problems. After students had successfully completed the practice problems, we discussed what a limit "really means", and how this is reflected in the computations they had been performing. By reversing the order of this particular lesson (practical *before* abstract), I found a noticeable decrease in student anxiety surrounding the topic of limits.

Educational Leadership

CMS Special Session on Math Education

I am a co-organizer of the upcoming Special Session *Teaching for Deeper Mathematical Engagement* at the 2016 CMS Winter Meeting in Niagara Falls, ON from Dec. 2-5, 2016 (https://cms.math.ca/Events/winter16/sessions_scientific#tdm). The aim of the session is to bring together educators from different institutions to discuss teaching practices and innovations that promote deeper learning in secondary and post-secondary Mathematics courses. My personal goal in organizing this session is to make connections in the Math Education community and to learn about the various initiatives taking place across Canada.

Teaching Workshop

I have developed a 90-minute workshop for graduate students, addressing how we can use language as a tool to increase student engagement. An outline of the workshop, including an annotated review of relevant literature and a detailed breakdown of the workshop's learning activities, has been submitted to a peer-reviewed journal and is available upon request.

On September 28th, 2016, I ran a version of this workshop at Western University, with 21 participants from the Departments of Mathematics, Applied Mathematics, and Statistics. The workshop began with a small group activity, where participants brainstormed ways of introducing new topics (e.g. implicit differentiation, continuous/discrete random variables) to first-year students using more accessible language. Next, we outlined research by Mesa & Chang concerning the roles of monogloss and heterogloss discursive voices in promoting and steering classroom discourse. Finally, we evaluated the helpfulness of various forms of feedback and discussed effective learning strategies which can be used to support struggling students.

After the workshop, participants were asked to complete an anonymous online survey to provide feedback. Results were very positive, and are summarized in Appendix B, along with a copy of a handout provided to workshop participants. A copy of the slides presented at the workshop are available online (www.carolinejunkins.com/teaching-workshop.html).

CMS Notes Article

In the September 2016 issue of Note of the Canadian Mathematical Society, I contributed a piece to the Education Notes column, titled *Changing the way we think about mathematical ability*. I was inspired to write this piece after reading the work of Carol Dweck on self-theories of intelligence, and how these theories apply to our views of mathematical ability as a fixed or malleable trait. As a result of this piece, I have had the opportunity to discuss important topics with math educators across Canada, ranging from the media's portrayal of math to the factors contributing to gender inequity within our discipline.

Professional Development

Connected Women in Mathematics Across Canada

(participated October 3-5, 2014)

This workshop at the Banff International Research Station was designed to to serve as a career starter retreat for young female mathematicians. During the workshop, "seasoned veterans" gave lectures on important skills including how to prepare a conference presentation, how to teach in a large classroom, and how to actively participate in the research community. I gave a short talk on my research interests, and received helpful feedback from the mentoring team. As a group, we discussed our career goals, our shared experiences, and the unique challenges faced by women in mathematics. For me, this workshop was an invaluable experience and gave me an opportunity to ask many important questions about the status of women in the mathematical community and how we can ensure that women are encouraged to participate in all areas of mathematics.

Advanced Teaching Program

(completed June 2016)

The Advanced Teaching Program is a hands-on 20-hour seminar offered through the Western Teaching Support Centre which is specifically designed for advanced graduate students and postdoctoral scholars. This seminar discusses topics including teaching strategically for maximum impact, developing and maintaining a culture of respect in your classroom, dealing with difficult students and authentically assessing student learning. As part of the program, I participated in two video-taped micro teaching sessions where I was able to reflect on my teaching and receive helpful feedback from peers and instructors. I also completed capstone project for the program which includes a literature review and workshop design incorporating various active learning techniques presented in the seminar.

Fall Perspectives on Teaching Conference

(participated August 30, 2016)

I participated in the Fall 2016 Perspectives on Teaching Conference, organized by the Western Teaching Support Centre. This particular conference focused on indigenising higher education, with a keynote session by Dr. Arini, Dean of the Faculty of Education and Social Work, Thompson Rivers University. Dr Arini spoke about her work on the ways in which universities can support the success of Indigenous students, and other students who are under-served in post-secondary education. This was followed by a panel discussion with faculty members from a variety of disciplines, sharing examples from their own classrooms of how they have incorporated indigenous knowledge and methodologies in their teaching in order to support student learning. The conference also included a session on Community Engaged Learning, describing the various ways in which instructors can incorporate community-based projects into their courses, adding value to both curriculum and community.

Math 1600 Activity - Intersections

- Complete this activity in groups of 4-5.
- Record your answers in one group workbook.
- Scrap paper, textbooks and/or lecture notes may be used as aids.
- **After completing each section, verify your group's answers with the TA before proceeding to the next section.**
- Please ensure that the name, student number, and lecture section of each group member is recorded on your workbook before handing it in at the end of tutorial.

Part A:

There are 3 possibilities for the intersection of two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in $\mathbb{R}^2$:

a) $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect in a unique line (i.e. $\mathcal{L}_1$ is equal to $\mathcal{L}_2$)

b) $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at a unique point $P = (x, y)$

c) $\mathcal{L}_1$ and $\mathcal{L}_2$ do not intersect (i.e. $\mathcal{L}_1$ is parallel to $\mathcal{L}_2$)

1. Suppose $\mathbf{n}_1$ is a normal vector for $\mathcal{L}_1$ and $\mathbf{n}_2$ is a normal vector for $\mathcal{L}_2$.
For each of a), b), and c), describe the relationship between $\mathbf{n}_1$ and $\mathbf{n}_2$.

2. Describe the 3 possibilities for the intersection of two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ in $\mathbb{R}^3$.

3. Suppose $\mathbf{n}_1$ is a normal vector for $\mathcal{P}_1$ and $\mathbf{n}_2$ is a normal vector for $\mathcal{P}_2$.
For each of these three possibilities, describe the relationship between $\mathbf{n}_1$ and $\mathbf{n}_2$.

Part B:

Suppose the planes $\mathcal{P}_1$ and $\mathcal{P}_2$ are defined by following equations:

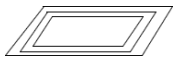
$$\begin{aligned} \mathcal{P}_1 : &\quad x + y - 2z = 3 \\ \mathcal{P}_2 : &\quad x - 2y + z = 4 \end{aligned}$$

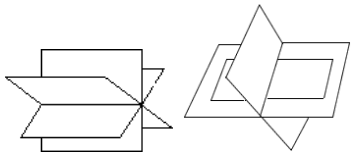
4. Which of your answers from **2.** applies to the intersection of these two planes?

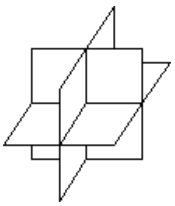
5. Provide an algebraic representation (e.g. coordinates of a point or equation of a line) for the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$, if such an intersection exists.

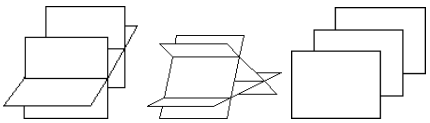
Part C:

There are 4 possibilities for the intersection of **three** planes $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ in $\mathbb{R}^3$:

a) The three planes intersect in a unique plane $\mathcal{Q}$ 

b) The three planes intersect at a unique line $\mathcal{L}$ 

c) The three planes intersect at a unique point P 

d) The three planes have no common intersection 

The intersection of three planes corresponds to a system of linear equations:

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

6. For each of first 3 possible situations a), b), and c), determine the number of **free variables** in the corresponding system of linear equations.

Suppose $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ are defined by following equations:

$$\begin{aligned} \mathcal{P}_1: &\quad x + y - 2z = 3 \\ \mathcal{P}_2: &\quad x - 2y + z = 4 \\ \mathcal{P}_3: &\quad 2x - y - z = 7 \end{aligned}$$

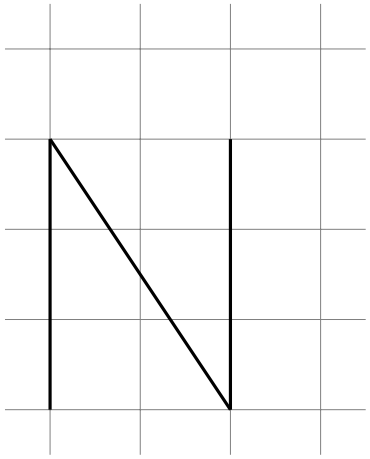
7. Using any method you'd like, determine which of the possible situations a), b), c), or d) describe the intersection of these three planes.

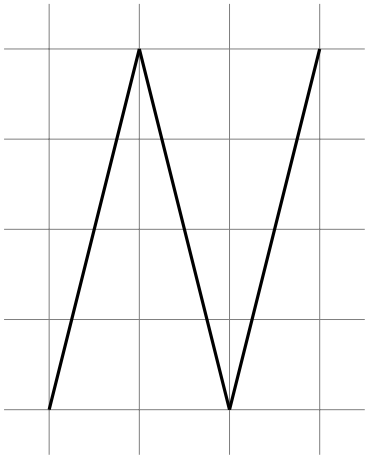
8. Provide an algebraic representation (e.g. coordinates of a point, equation of a line, or equation of a plane) for the intersection of $\mathcal{P}_1, \mathcal{P}_2$, and $\mathcal{P}_3$, if such an intersection exists.

Activity adapted from: Inquiry-Oriented Instructional Materials for Linear Algebra (IOLA) http://iola.math.vt.edu

Math 1600 Activity - Linear Transformations

- Complete this activity in groups of 4-5.
- Record your answers in one group workbook.
- Scrap paper, textbooks and/or lecture notes may be used as aids.
- **After completing each section, verify your group's answers with the TA before proceeding to the next section.**
- Please ensure that the name, student number, and lecture section of each group member is recorded in your workbook before handing it in at the end of tutorial.





Suppose the "N" on the left is written in regular 12-point font and that the "N" on the right is written in *italics* in 16-point font.

Part A:

Let T be the transformation which takes the "N" on the left to the "N" on the right.

1. What is the domain of T?
2. What is the codomain of T?
3. Describe the action T has on the letter "N". Try to be as specific as possible.

Let A be a matrix which represents the transformation T. This means that for a vector x in the domain of T and a vector y in the codomain of T, $T(x) = y$ if and only if $Ax = y$.

4. What is the dimension of A?
5. What type of input-output pairs $\{x, y\}$ can be used to determine the matrix A?
6. How many input-output pairs are required to determine A?
7. Using the graphs above, provide a sufficient set of input-output pairs.
   Hint: It may be useful to place the origin at the bottom left corner of the "N".
8. Use these input-output pairs to compute the matrix A.

Part B:

Consider the following approach to determining the matrix A described in Part A:

> In order to find the matrix A, we are going to find a matrix S that makes the "N" taller (from 12- point to 16-point), find a matrix L that italicizes the taller "N", and the combination of those will be the desired matrix A.

1. Determine the matrices S and L described in this approach.
2. How could you "combine" these matrices to obtain A?
3. If you "combine" these matrices in a different order, do you still obtain A? Provide justification for your yes/no answer.

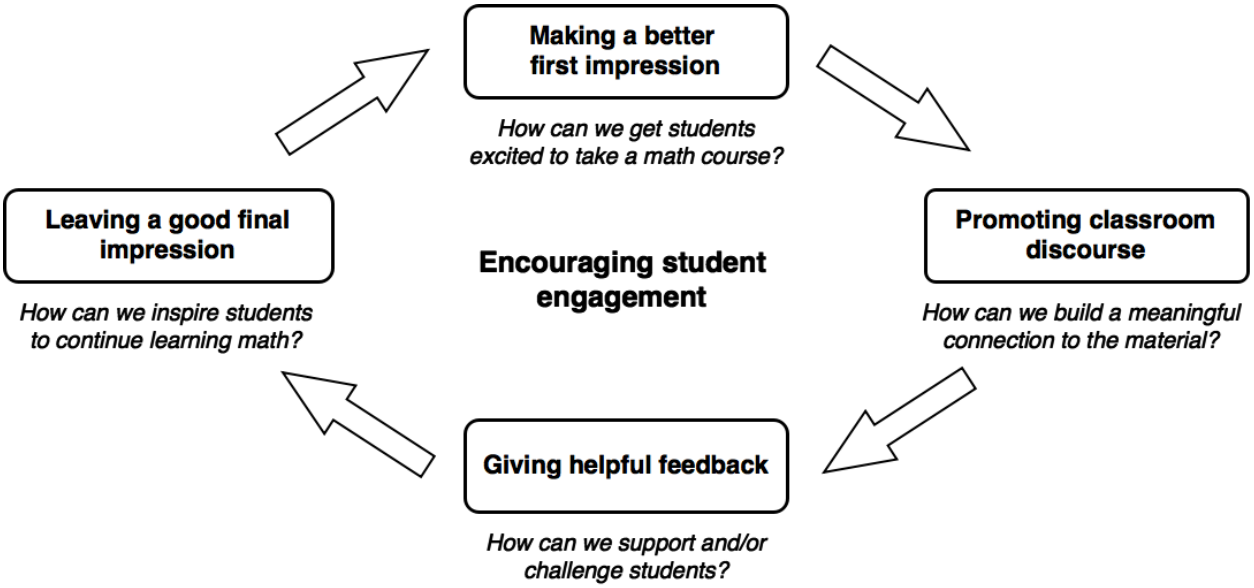
Part C:

1. Determine the matrix C which transforms the "N" on the right back to the "N" on the left.
2. Describe the relationship between C and the matrix A from Part A.
3. Describe the relationship between C and the matrices S and L from Part B.
4. Describe the relationship between C and the linear transformation T.

Activity adapted from: Unit 2: Tasks 1-4, Inquiry-Oriented Instructional Materials for Linear Algebra (IOLA) http://iola.math.vt.edu

Encouraging student engagement through small changes to the language we use in a first-year math course

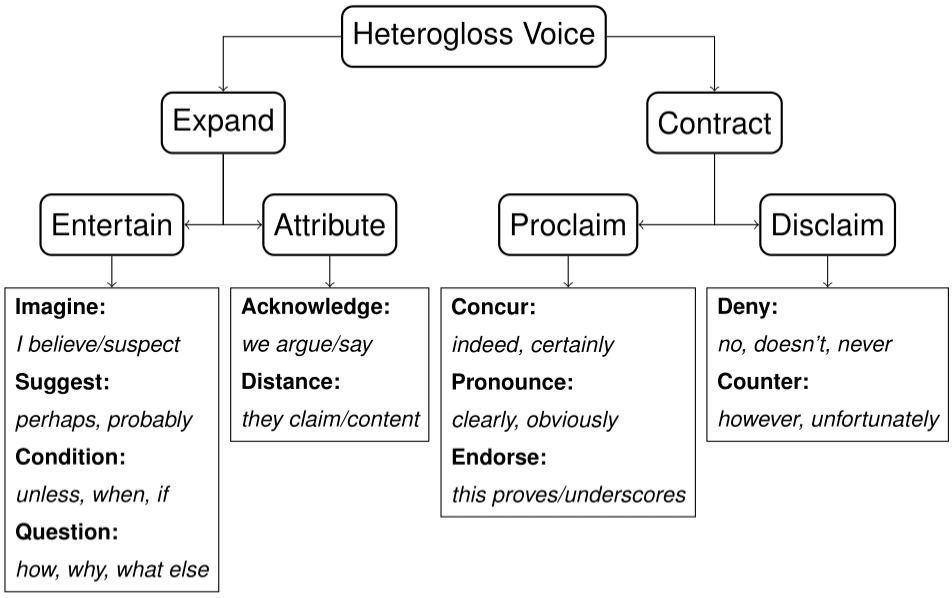
Caroline Junkins, UWO Department of Mathematics, 2016



Promoting Classroom Discourse

Monogloss voice is used to present the standardized forms of mathematical constructs. It is neccessary to ensure consistency and rigour, but can sound impersonal or report-like.

Heterogloss voice is used to convey the diverse expressions of mathematical meaning. It grounds a proposition in the personal subjectivity of the speaker and engages the listener at different levels. Heterogloss voice can be used to promote and steer conversations with students.



Adapted from Mesa, V., & Chang, P. (2010). The language of engagement in two highly interactive undergraduate mathematics classrooms, *Linguistics and Education, 21* (2), 83-100.

Giving Helpful Feedback

Statements of Caring (neutral)

- ➔ Let's stay in contact to discuss how you are doing with the material.
- ➔ You are a talented student. I am glad you are taking my course.
- ➔ I can see that you are working hard to learn these concepts.
- ➔ I understand that you probably have a lot of work in your other courses right now.

Statements of caring are a typical component of positive feedback and can help build the student-instructor relationship. However, this is somewhat orthogonal to the primary goal of feedback, which is to bridge the gap between current understanding and expected learning outcomes.

Comfort-Oriented Statements (NOT helpful)

- ➔ Not everyone is a "math person"; your strengths just lie in other subjects.
- ➔ I'm going to give you some easier questions to work on instead so that you can get more comfortable with those skills.
- ➔ I won't call on you directly in class because I don't want to add extra pressure by putting you on the spot.
- ➔ Maybe you just need a little more time to finish the assignment; you can have until Monday.

When students receive comfort-oriented feedback, they perceive their instructor as having lower expectations and investment. They feel significantly less encouraged and motivated, and expect to receive a lower final grade in the course.

Strategy-Oriented Statements (MOST helpful)

- ➔ I want you to consider working with a tutor or with another student from class.
- ➔ I will make a point of calling on you more in class to ensure you are staying on track.
- ➔ I'm going to give you some more challenging questions so that you can learn to apply these skills to new situations.
- ➔ If you break down the assignment into manageable pieces and work on one each day, you will be able to complete it on time.

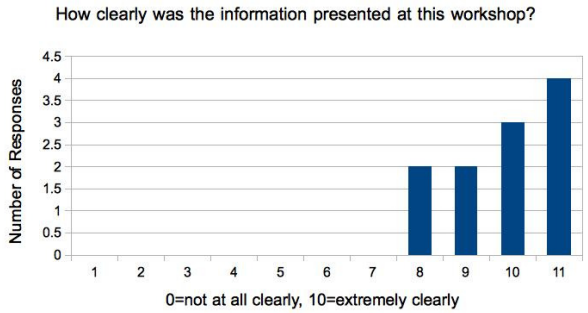
Students who receive strategy-oriented feedback have more positive perceptions of their instructor's expectations and investment than those who receive only statements of caring. The use of effort management strategies is correlated to motivation and is a significant predictor of achievement.

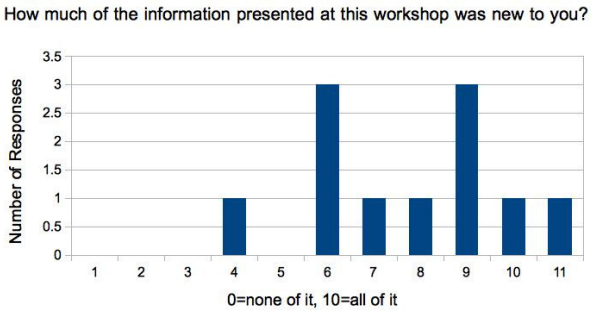
Pokay, P., & Blumenfeld, P.C. (1990). Predicting achievement early and late in the semester: The role of motivation and use of learning strategies, *Journal of Educational Psychology, 82* (1), 45-50.

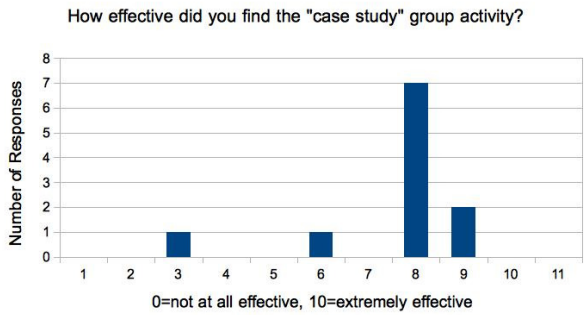
Rattan, A., Good, C., & Dweck, C.S. (2012). "It's ok — Not everyone can be good at math": Instructors with an entity theory comfort (and demotivate) students, *Journal of Experimental Social Psychology, 48* (3), 731-737.

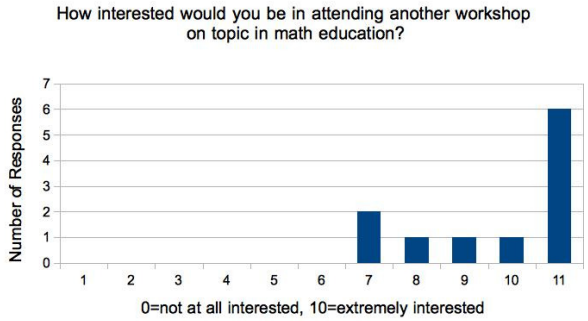
Results of the Workshop Survey

Out of the 21 graduate students who participated in the Teaching Workshop *The language we use to teach mathematics*, 11 completed a short anonymous online survey.







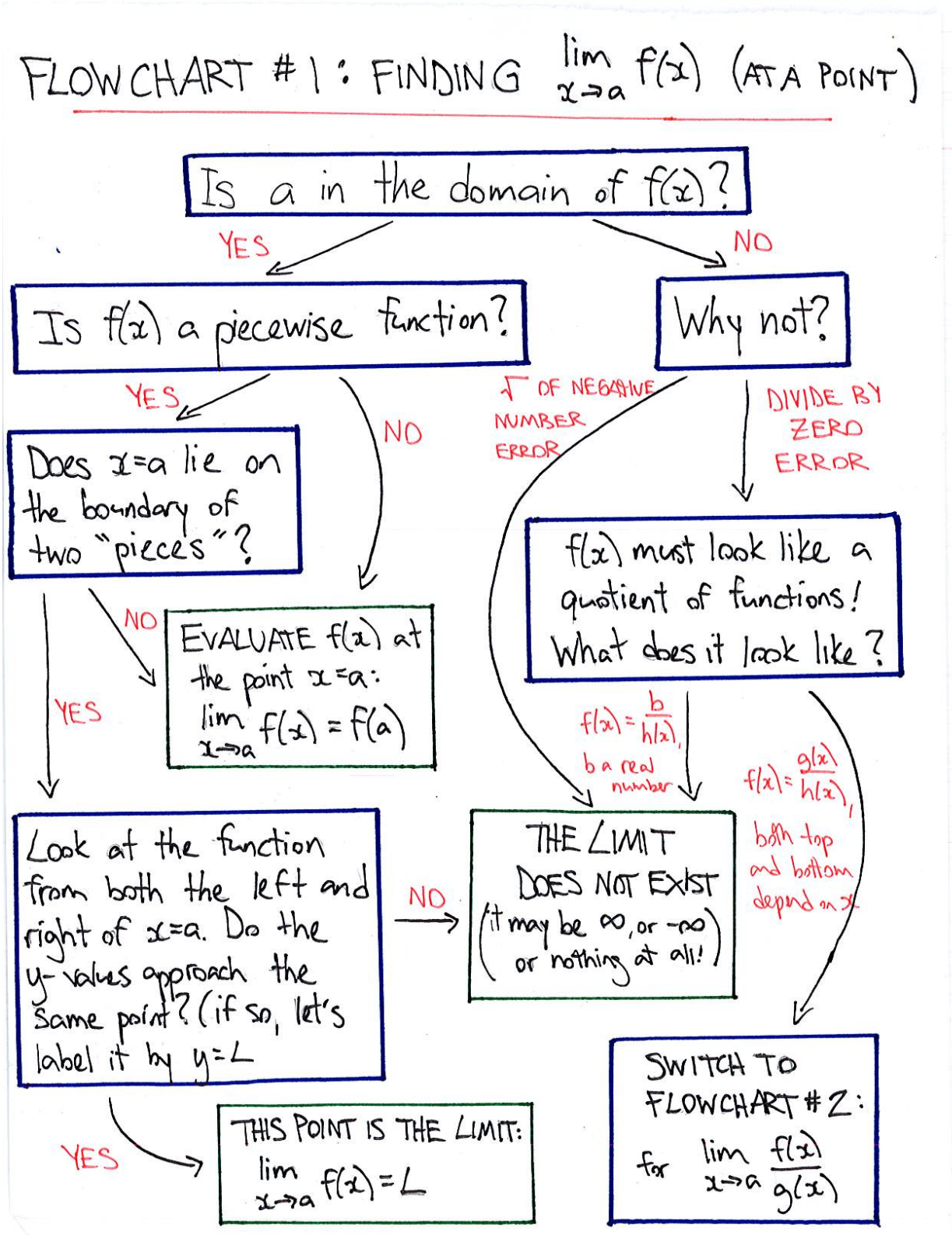


Which topic presented in the workshop did you find most interesting?

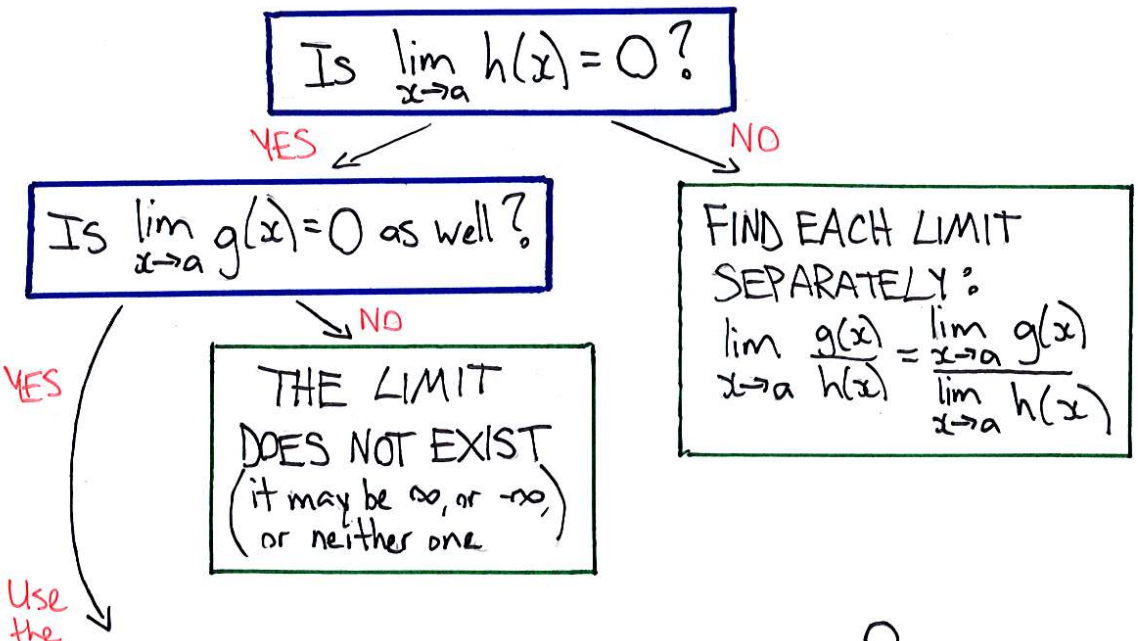
1. Engaging students with mathematical ideas (Heterogloss voice): 8 respondents
2. Using accessible language for new students (Case study): 2 respondents
3. Providing feedback and support to students (Learning strategies): 1 respondent

Comments/Suggestions

We would like to have more case studies.
Very enjoyable.
In future more advance topics may covered.
Loved the example for feedback where you might be hindering students by trying to comfort them in saying "it's okay not everyone gets it." Hadn't thought of that before. Wished the seminar was longer actually! (Would maybe need to be on a weekend if it was long to make more accessible). Caroline was very well spoken and clear and a great presenter :)
I think mathematics is one of the hardest subjects to learn/teach, and many students (or even general audiences) struggle with the basics. Asking ourselves why that is is the first step towards an answer. Thanks for asking the right questions; I hope we will have the occasion to ask many others :)
The quotations we put stickers on should have been a little less clear cut; the good ones were obviously good and the bad ones obviously bad, so there wasn't much to think about.


FLOWCHART #1: FINDING $\lim_{x\to a} f(x)$ (AT A POINT)
Is a in the domain of f(x)?
YES
NO
Is f(x) a piecewise function?
Why not?
YES
NO
√ OF NEGATIVE NUMBER ERROR
DIVIDE BY ZERO ERROR
Does x=a lie on the boundary of two "pieces"?
f(x) must look like a quotient of functions! What does it look like?
NO
YES
EVALUATE f(x) at the point x=a: $\lim_{x\to a} f(x) = f(a)$
$f(x) = \frac{b}{h(x)}$, b a real number
$f(x) = \frac{g(x)}{h(x)}$, both top and bottom depend on x
Look at the function from both the left and right of x=a. Do the y-values approach the same point? (if so, let's label it by y=L
NO
THE LIMIT DOES NOT EXIST (it may be ∞, or -∞ or nothing at all!)
YES
THIS POINT IS THE LIMIT: $\lim_{x\to a} f(x) = L$
SWITCH TO FLOWCHART #2: for $\lim_{x\to a} \frac{f(x)}{g(x)}$

FLOWCHART #2 : FINDING $\lim_{x\to a} \frac{g(x)}{h(x)}$ (AT A POINT)



STRATEGY FOR INDETERMINATE FORM $\frac{0}{0}$:

1. Replace $\frac{g(x)}{h(x)}$ with another function (let's call it $p(x)$) which takes on the same values everywhere except $x = a$.

   HOW? → Factor both numerator and denominator, then cancel any common factors to get $p(x)$.

   ↘ **If there is a "$\sqrt{\ }$":** Multiply by the conjugate, then expand, simplify, and cancel common factors.

2. Evaluate the limit of this new function as x approaches a. That is, $\lim_{x\to a} \frac{g(x)}{h(x)} = \lim_{x\to a} p(x)$

   HOW? To find this new limit, go back to Flowchart #1.