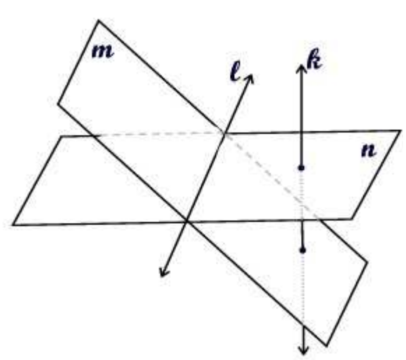
Math 1600 Activity - Fundamental Subspaces

- Complete this activity in groups of 4-5.
- Record your answers in one group workbook.
- Scrap paper, textbooks and/or lecture notes may be used as aids.
- **After completing each section, verify your group's answers with the TA before proceeding to the next section.**
- Please ensure that the name, student number, and lecture section of each group member is recorded in your workbook before handing it in at the end of tutorial.

For the entire activity, we consider the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 6 & -3 \end{bmatrix}$.

Part A:

1. Find a basis for row(A) and use this basis produce a geometric description of row(A) (i.e. equation of a line/plane/hyperplane, or coordinates of a point).
2. Find a basis for null(A) and use this basis to produce a geometric description of null(A) (i.e. equation of a line/plane/hyperplane, or coordinates of a point).
3. Choose a nonzero vector $\mathbf{u}$ in row(A) and a nonzero vector $\mathbf{v}$ in null(A). Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.
4. Will you get the same answer for $\mathbf{u} \cdot \mathbf{v}$ given *any* $\mathbf{u}$ in row(A) and $\mathbf{v}$ in null(A)?
5. In the picture below, which pair of geometric objects would describe the relationship between row(A) and null(A)?



Part B:

1. Graph the subspaces $\text{col}(A) = \text{row}(A^T)$ and $\text{null}(A^T)$ in the xy-plane.
2. Describe (in words) the relationship between a vector $\mathbf{u}$ in $\text{col}(A)$ and a vector $\mathbf{v}$ in $\text{null}(A^T)$.

For a subspace W of $\mathbb{R}^n$, we define the **orthogonal complement** $W^\perp$ of W by

$$W^\perp = \{v \in R^n \text{ such that } \mathbf{v} \cdot \mathbf{w} = 0 \text{ for all } \mathbf{w} \text{ in } W\}$$

3. If $W = \text{row}(A)$, what is $W^\perp$? If $V = \text{col}(A)$, what is $V^\perp$?

Let W be a subspace of $\mathbb{R}^n$. If $\mathcal{B}_1$ is a basis of W and $\mathcal{B}_2$ is a subspace of $W^\perp$, then $\mathcal{B}_1 \cup \mathcal{B}_2$ is a basis for $\mathbb{R}^n$. In particular, every vector $\mathbf{v}$ in $\mathbb{R}^n$ can be written as $\mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2$, where $\mathbf{w}_1$ is in W and $\mathbf{w}_2$ is in $W^\perp$.

4. Let $\mathbf{v} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$. Find $\mathbf{w}_1$ in $\text{col}(A)$ and $\mathbf{w}_2$ in $\text{null}(A^T)$ such that $\mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2$.
5. Let $\mathbf{v} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$. Find $\mathbf{w}_1$ in $\text{row}(A)$ and $\mathbf{w}_2$ in $\text{null}(A)$ such that $\mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2$.