Math 1600 Activity - Intersections

- Complete this activity in groups of 4-5.
- Record your answers in one group workbook.
- Scrap paper, textbooks and/or lecture notes may be used as aids.
- **After completing each section, verify your group's answers with the TA before proceeding to the next section.**
- Please ensure that the name, student number, and lecture section of each group member is recorded on your workbook before handing it in at the end of tutorial.

Part A:

There are 3 possibilities for the intersection of two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in $\mathbb{R}^2$:

a) $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect in a unique line (i.e. $\mathcal{L}_1$ is equal to $\mathcal{L}_2$)

b) $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at a unique point $P = (x, y)$

c) $\mathcal{L}_1$ and $\mathcal{L}_2$ do not intersect (i.e. $\mathcal{L}_1$ is parallel to $\mathcal{L}_2$)

1. Suppose $\mathbf{n}_1$ is a normal vector for $\mathcal{L}_1$ and $\mathbf{n}_2$ is a normal vector for $\mathcal{L}_2$.
For each of a), b), and c), describe the relationship between $\mathbf{n}_1$ and $\mathbf{n}_2$.

2. Describe the 3 possibilities for the intersection of two planes $\mathcal{P}_1$ and $\mathcal{P}_2$ in $\mathbb{R}^3$.

3. Suppose $\mathbf{n}_1$ is a normal vector for $\mathcal{P}_1$ and $\mathbf{n}_2$ is a normal vector for $\mathcal{P}_2$.
For each of these three possibilities, describe the relationship between $\mathbf{n}_1$ and $\mathbf{n}_2$.

Part B:

Suppose the planes $\mathcal{P}_1$ and $\mathcal{P}_2$ are defined by following equations:

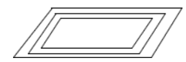
$$\begin{aligned} \mathcal{P}_1 : \quad & x + y - 2z = 3 \\ \mathcal{P}_2 : \quad & x - 2y + z = 4 \end{aligned}$$

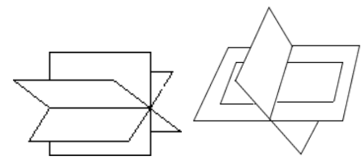
4. Which of your answers from **2.** applies to the intersection of these two planes?

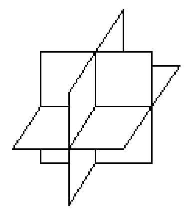
5. Provide an algebraic representation (e.g. coordinates of a point or equation of a line) for the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$, if such an intersection exists.

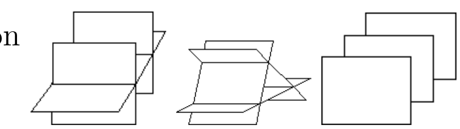
Part C:

There are 4 possibilities for the intersection of **three** planes $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ in $\mathbb{R}^3$:

a) The three planes intersect in a unique plane $\mathcal{Q}$ 

b) The three planes intersect at a unique line $\mathcal{L}$ 

c) The three planes intersect at a unique point P 

d) The three planes have no common intersection 

The intersection of three planes corresponds to a system of linear equations:

$$
\begin{aligned}
a_1x + b_1y + c_1z &= d_1 \\
a_2x + b_2y + c_2z &= d_2 \\
a_3x + b_3y + c_3z &= d_3
\end{aligned}
$$

6. For each of first 3 possible situations a), b), and c), determine the number of **free variables** in the corresponding system of linear equations.

Suppose $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ are defined by following equations:

$$
\begin{aligned}
\mathcal{P}_1 &: \quad x + y - 2z = 3 \\
\mathcal{P}_2 &: \quad x - 2y + z = 4 \\
\mathcal{P}_3 &: \quad 2x - y - z = 7
\end{aligned}
$$

7. Using any method you'd like, determine which of the possible situations a), b), c), or d) describe the intersection of these three planes.

8. Provide an algebraic representation (e.g. coordinates of a point, equation of a line, or equation of a plane) for the intersection of $\mathcal{P}_1, \mathcal{P}_2$, and $\mathcal{P}_3$, if such an intersection exists.

Activity adapted from: Inquiry-Oriented Instructional Materials for Linear Algebra (IOLA) http://iola.math.vt.edu